## Volume of Mixing in the Liquid Bismuth-Lead System

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**Synopsis.** The results obtained in determining the volume of mixing  $v^m$  in the liquid Bi-Pb system at 660 K using the pycnometer method indicate the compound Pb<sub>3</sub>Bi<sub>2</sub> at the composition 0.6. The shifting of the slight positive  $v_{\max}^m$  to the composition 0.4 is attributed to a nonlinear Wagner effect.

The behavior of liquid metallic systems has not yet been thoroughly explored. The theoretical relationships between the thermodynamic functions of metallic mixtures were derived for simple models e.g, the theory by Prigogine<sup>1)</sup> which gives the relation of the volume of mixing to the enthalpy of mixing but could not illustrate the behavior of mixing in the liquid bismuth-lead system<sup>2)</sup> or other of complicated behaviors.

In the present investigation we report the results obtained in determining the volume of mixing in the liquid Bi-Pb system at 660 K by using the pycnometer method since these results could shed light on the behavior of mixing in the Bi-Pb system and other related binary ones.

## **Experimental**

The metal used were of extra purity grade (E. Merck Company). The experimental procedures adopted were similar to those used earlier.<sup>3)</sup> The apparatus employed was a slightly modified version of the type used by Wittig and Huber.<sup>3)</sup> Argon gas was used as a protector in order to avoid oxidation.

**Theory and Procedure.** The additional behavior  $y^{ad}$  is given by

$$y^{ad} = y_1^{\circ} + (y_2^{\circ} - y_1^{\circ}) x_2 \tag{1}$$

where  $y_1^{\circ}$  and  $y_2^{\circ}$  are the values of the molar quantities of the components 1 and 2, respectively and  $x_2$  the mole fraction of the component 2. The form function of the excess function,  $X^y$  is given through the equation

$$X^{y}(x_{2}) = \frac{y^{e}}{x_{2}(1-x_{2})}$$
 (2)

y<sup>e</sup> refers to the excess quantity. The excess volume and partial excess volumes can be expressed as polynomial functions:

Table 1. Experimentally Determined Densities (×10<sup>-3</sup>) for Liquid Lead-Bismuth at 660K in kg m<sup>-3</sup>.

Estimated Standard Deviations in Parenthesis

$x_2$	d	$x_2$	d
0	9.9318(21)	0.6	10.2994(15)
0.1	9.9839(20)	0.7	10.3656(21)
0.2	10.0404(20)	0.8	10.4417(20)
0.3	10.0967(20)	0.9	10.5148(21)
0.4	10.1643(21)	1.0	10.5882(20)
0.5	10.2305(20)		, ,

$$v^{e}(x_{2}) = \sum_{k,l} a_{kl} x_{2}^{k} = \sum_{k,l} a_{kl} x_{2}^{k} (1 - x_{2})^{l}$$
(3)

$$v_1^e(x_2) = \sum_{k,l} (1-k) a_{kl} x_2^k$$
 (4)

$$v_2^{e}(x_2) = \sum_{k,l} (1-k) a_{kl} x_2^{k} + \sum_{k,l} k a_{kl} x_2^{k-l}.$$
 (5)

The density d has been computed at the temperature of measurement  $\vartheta$  from the mass m and the volume of the pycnometer  $V_{py}$  as follows:

$$d = m/V^{22.9} \cdot [1 + 3a (\vartheta - 22.9)], \tag{6}$$

where 22.9 °C is the calibration temperature of the pycnometer and a the thermal coefficient of silica glass which is given by

$$a_{\text{SiO}_2} = 0.59 \times 10^{-6} \,\text{K}^{-1}.$$
 (7)

## Results and Discussion

Results of the density determinations are presented in Table 1. The results have been least squares fitted by the following polynomial expression:

$$d(x_2) = 9.9297 + 0.5003 x_2 + 0.2332 x_2^2 - 0.0769 x_3^3.$$
 (8)

The standard deviation is 0.02% for a single measurement.

In Table 2 are collected the values of the average molar volume of the mixture of Bi-Pb as found in the present study. The results have been least squares fitted by a polynomial expression with a standard deviation of 0.012% for a single measurement. The polynomial expression is given by

$$v(x_2) = 21.0458 - 1.2445 x_2 - 0.4103 x_2^2 + 0.1805 x_3^3$$
 (9)

Table 3 represents the values of the integral and partial molar volume of mixing as determined in the present study. The volume of mixing can be calculated according to the following equation:

$$v^{\rm m} = v - v^{\rm ad} = v - v_1^{\rm o} + x_2 (v_2^{\rm o} - v_1^{\rm o}) \tag{10}$$

Table 2. Average Molar Volumes (×10<sup>6</sup>) for the Mixture Bi-Pb in m<sup>3</sup> mol<sup>-1</sup>. Estimated Standard Deviations in Parenthesis

$x_2$	v	$x_2$	v
0	21.0483(25)	0.6	20.1929(26)
0.1	20.9200(26)	0.7	20.0378(23)
0.2	20.7844(25)	0.8	19.8774(25)
0.3	20.6429(26)	0.9	19.7229(25)
0.4	20.4913(26)	1.0	19.5737(23)
0.5	20.3410(25)		, ,

Table 3. Integral and Partial Molar Volume of Mixing (×10<sup>6</sup>) for the Liquid Bi-Pb System at 660 K in m<sup>3</sup> mol<sup>-1</sup>. Estimated Standard Deviations in Parenthesis

<i>x</i> <sub>2</sub>	$v^{m}$	$v_1^{\rm m}$	$v_2^{\mathrm{m}}$	
0	0.0022(22)	0.0000	0.2297	
0.1	0.0213(24)	0.0037	0.1569	
0.2	0.0285(23)	0.0135	0.1009	
0.3	0.0348(24)	0.0272	0.0595	
0.4	0.0353(24)	0.0425	0.0308	
0.5	0.0325(23)	0.0575	0.0123	
0.6	0.0267(23)	0.0697	0.0021	
0.7	0.0240(24)	0.0772	-0.0019	
0.8	0.0112(23)	0.0778	-0.0022	
0.9	0.0035(24)	0.0692	-0.0008	
1.0	0.0024(24)	0.0493	0.0000	

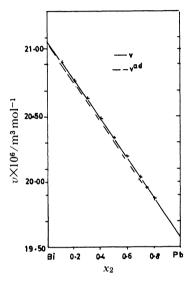


Fig. 1. Average molar volume, v, and additional volume,  $v^{ad}$ , for the mixture Bi-Pb.

where  $v_1^{\circ}$  and  $v_2^{\circ}$  are the molar volumes of the pure components 1 and 2, respectively. Figure 1 shows the average molar volume of the mixture, v and the additional volume  $v^{\text{ad}}$ . For the volume of mixing, the following polynomial expression is derived:

$$v^{\rm m} = 0.2298 \ x_2 - 0.4103 \ x_2^2 + 0.1805 \ x_2^3 \tag{11}$$

The partial molar volume of mixing of bismuth and lead are given through the following equations:

$$v_1^{\rm m} = 0.04103 \ x_2^2 - 0.3610 \ x_2^3 \tag{12}$$

$$v_2^{\rm m} = 0.2298 - 0.8206 \ x_2 + 0.9518 \ x_2^2 - 0.3610 \ x_2^3$$
 (13)

As illustrated in Table 3, the estimated standard deviation (esd.) is not larger than expected from the pycnometer measurement uncertainties,  $2.2\times10^{-9}$ ,  $2.4\times10^{-9}$  m³ mol<sup>-1</sup>. The plot of  $v^{\rm m}$  versus the mole fraction  $x_2$  is shown in Fig. 2. It is clearly observable that  $v^{\rm m}$  takes slight positive values. The curve of  $v^{\rm m}$  against  $x_2$  and those of  $v^{\rm m}_{\rm Bi}$  and  $v^{\rm m}_{\rm Pb}$  point out to the existence of a compound. This was also recognized from the behavior of the enthalpy and free enthalpy in this system.<sup>4)</sup>

According to the Wagner effect,5) the compound will

Table 4. The Form Function of the Volume of Mixing (×10<sup>6</sup>) for the Liquid Bi-Pb System at 660 K in m<sup>3</sup> mol<sup>-1</sup>. Estimated Standred Deviations in Parenthesis

$x_2$	Χ <sup>v</sup>	<i>x</i> <sub>2</sub>	$X^{v}$
0	0.2323(26)	0.6	0.1194(20)
0.1	0.2137(20)	0.7	0.1059(25)
0.2	0.1912(20)	0.8	0.0833(20)
0.3	0.1781(25)	0.9	0.0698(25)
0.4	0.1597(22)	1.0	$0.0492^{a}$
0.5	0.1372(23)		

a) Calculated value.

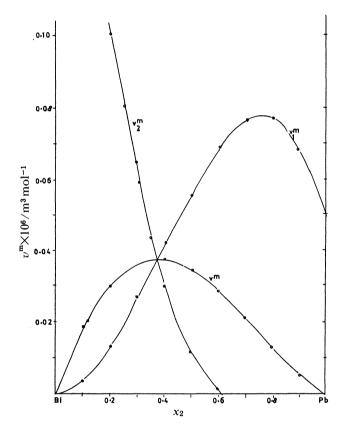


Fig. 2. Integral and partial molar volume of mixing for the liquid Bi-Pb system at 660 K.

be formed as follows:  $3 \text{ Pb}+2 \text{ Bi}=\text{Pb}_3 \text{Bi}_2$ . The composition of the compound and also that of the maximum of the volume of mixing,  $v_{\text{max}}^m$ , will be: 1-2/(2+3)=0.6. However, the Wagner effect requires sufficiently large difference in electronegativity between the acceptor (Bi) and the donor (Pb). In fact, the electronegativity difference between Bi and Pb (0.1-0.12) is not large enough. This could be a reason for the shifting of the maximum of the volume of mixing from the composition 0.6 to 0.4 (Fig. 2). This means that we have here to deal with a nonlinear Wagner effect as found in the systems  $\text{Zn-Sb}^{7}$  and  $\text{Cd-Bi}.^{8}$  A second possibility of the existence of a compound at the composition 0.4 (the composition of  $v_{\text{max}}^m$ ) still remains under investigation.

In Table 4 are listed the values of the form function of the volume of mixing which is given by the follow-

ing linear function:

$$X^{\mathsf{v}} = 0.2298 - 0.1805 \ x_2 \tag{14}$$

As given in Table 4, the esd. ranged from  $2.0 \times 10^{-9}$  to  $2.6 \times 10^{-9}$  m<sup>3</sup> mol<sup>-1</sup>.

## References

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